

## SOLUTION TO QUIZ 6

MATH 241

Write the following ODE in a Sturm-Liouville problem in self-adjoint form, and write down the orthogonal relation between any two eigenfunctions  $y_1(x), y_2(x)$  in terms of an integral.

$$y'' + (\sin x)y' + \lambda y = 0$$

$$y(0) = 0, y(4) = 0$$

*Proof.* The integral factor is  $e^{\int \sin x dx} = e^{-\cos x}$ , multiply this to the equation and get  $e^{-\cos x}y'' + e^{-\cos x}(\sin x)y' + \lambda e^{-\cos x}y = 0$ , in the self-adjoint form as

$$(e^{-\cos x}y')' + \lambda e^{-\cos x}y$$

Therefore the eigenfunctions are orthogonal with respect to the weight function  $e^{-\cos x}$ , and explicitly, for any two eigenfunctions  $y_1(x), y_2(x)$ , we have

$$\int_0^4 e^{-\cos x}y_1(x)y_2(x)dx = 0$$

□